

Monogamy of Quantum Discord by Multipartite Correlations

H. C. Braga,* C. C. Rulli, Thiago R. de Oliveira, and M. S. Sarandy

Instituto de Física, Universidade Federal Fluminense,

Av. Gal. Milton Tavares de Souza s/n, Gragoatá, 24210-346, Niterói, RJ, Brazil.

(Dated: July 12, 2012)

We introduce a monogamy inequality for pairwise quantum correlations, which implies that the sum of pairwise quantum correlations is upper limited by the amount of multipartite quantum correlations as measured by the global quantum discord. This monogamy bound holds for either pure or mixed quantum states provided that the conditional mutual information does not increase under measurement. We illustrate the monogamy behavior for multipartite pure states with Schmidt decomposition as well as for W-GHZ mixed states. As a by-product, we apply the monogamy bound to investigate residual multipartite correlations.

PACS numbers: 03.67.-a, 03.67.Mn, 03.65.Ud

Quantum discord (QD) [1], has recently been identified as a general resource in quantum information protocols (see, e.g., Refs. [2, 3]). In quantum computation, it has been conjectured as the origin of speed up in the deterministic quantum computation with one qubit (DQC1) mixed-state model [4]. Moreover, a fundamental role has been attributed to QD in tasks such as quantum locking [5] and quantum state discrimination [6]. Besides quantum protocols, remarkable applications of QD have also been found in the characterization of quantum phase transitions [7] and in the description of quantum dynamics under decoherence [8].

In order to use quantum correlations (as provided by QD) as a resource, we are faced with the problem of their distribution throughout a multipartite state. In this context, a monogamous behavior may reveal important information about the structure of quantum correlations. Moreover, monogamy is the essential feature allowing for security in quantum key distribution [9]. In general grounds, it has been investigated whether any given measure \mathcal{Q} of bipartite quantum correlation can obey a monogamy bound in an arbitrary tripartite quantum system ABC . A measure \mathcal{Q} has typically been defined as monogamous if it follows the inequality

$$\mathcal{Q}(A, BC) \geq \mathcal{Q}(A, B) + \mathcal{Q}(A, C). \quad (1)$$

Therefore, given a fixed value for the quantum correlation $\mathcal{Q}(A, BC)$ between subsystem A and the remaining part BC of the system, then A cannot freely correlate with the individual subsystems B and C . Such monogamy bound is obeyed by certain entanglement measures, which is indeed the origin of *tangle* [10] as a measure of genuine multipartite entanglement. Whether or not monogamy can be obtained for a quantum correlation measure in arbitrary (pure or mixed) quantum states poses therefore as a further challenge. However, many attempts turned out to find a polygamous behavior for QD [11–15]. More generally, such a negative answer has been obtained for any sensible measure \mathcal{Q} of quantum correlations [16] (even though a function of \mathcal{Q} may be able to

yield a state-dependent monogamy relationship [17]).

The violation of the monogamy inequality (1) means that $\mathcal{Q}(A, BC)$ is unable in general to set a limit for the sum of pairwise correlations. However, it does *not* imply that subsystems can freely correlate. More specifically, it is possible to restore monogamy if we can find out a physical quantity that is able to provide a bound for the sum of pairwise correlations. *Here, we will investigate this problem by exploring such a route, providing a monogamy bound that holds for either pure or mixed states in a multipartite system composed of a number N of subsystems.* In order to achieve this aim, we will show that a limit for the sum of pairwise quantum correlations is provided by a multipartite extension of QD, named as *global quantum discord* (GQD), which has been proposed in Ref. [18]. In the case of tripartite states, a monogamy bound will then be obtained by replacing $\mathcal{Q}(A, BC)$ for the GQD, denoted by $\mathcal{D}(A : B : C)$, yielding

$$\mathcal{D}(A : B : C) \geq \mathcal{D}(A : B) + \mathcal{D}(A : C). \quad (2)$$

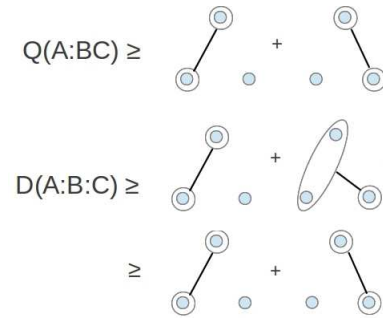


FIG. 1. (Color online) Schematic description of tripartite monogamy structure for $\mathcal{Q}(A, BC)$ and $\mathcal{D}(A : B : C)$.

A schematic description comparing the monogamy bounds for $\mathcal{Q}(A, BC)$ and $\mathcal{D}(A : B : C)$ is displayed in Fig. 1. The validity of inequality (2) will be shown for all quantum states whose conditional mutual information

does not increase under measurement. Moreover, we will show that GQD allows for the extension of Eq. (2) for the case of N subsystems. We observe that the maximum of GQD typically increases with N , which accounts for the fact that pairwise correlations is less monogamous than entanglement. On the other hand, as will be illustrated in several examples, GQD is sufficiently limited to provide a useful constraint for the sum of pairwise QD. As an application, we will use Eq. (2) to propose a natural measure of residual multipartite quantum correlations.

Quantum discord and its multipartite extension. — QD, which has been introduced in Ref. [1] and provides a measure of the quantumness of correlations, can be suitably defined as the minimal loss of total correlation after a non-selective measurement [19]. Indeed, consider a bipartite system AB composed of subsystems A and B . Denoting by $\hat{\rho}_{AB}$ the density operator of AB and by $\hat{\rho}_A$ and $\hat{\rho}_B$ the density operator of parts A and B , respectively, the total correlation between A and B is measured by the quantum mutual information [20]

$$I(\hat{\rho}_{AB}) = S(\hat{\rho}_A) - S(\hat{\rho}_A|\hat{\rho}_B), \quad (3)$$

where $S(\hat{\rho}_A) = -\text{Tr}\hat{\rho}_A \log_2 \hat{\rho}_A$ is the von Neumann entropy for A and $S(\hat{\rho}_A|\hat{\rho}_B) = S(\hat{\rho}_{AB}) - S(\hat{\rho}_B)$ is the entropy of A conditional on B . By operating on subsystem B , QD can then be expressed as $\mathcal{D}(A|B) = \min_{\{\hat{\Pi}_B^j\}} [I(\hat{\rho}_{AB}) - I(\Phi_B(\hat{\rho}_{AB}))]$, where $\Phi_B(\hat{\rho}_{AB})$ denotes a non-selective measurement $\{\hat{\Pi}_B^j\}$ on part B of $\hat{\rho}_{AB}$, which yields $\Phi_B(\hat{\rho}_{AB}) = \sum_j (\hat{1}_A \otimes \hat{\Pi}_B^j) \hat{\rho}_{AB} (\hat{1}_A \otimes \hat{\Pi}_B^j)$. Note that this definition is asymmetric with respect to measurement on A or B . In particular, a strictly classical state requires both $\mathcal{D}(A|B) = 0$ and $\mathcal{D}(B|A) = 0$. Indeed, this corresponds to a density operator $\hat{\rho}_{AB} = \sum_{i,j} p_{ij} |i\rangle\langle i| \otimes |j\rangle\langle j|$, where p_{ij} is a joint probability distribution and the sets $\{|i\rangle\}$ and $\{|j\rangle\}$ constitute orthonormal bases for the systems A and B , respectively. Such strictly classical states can also be identified by a single measure, which is the symmetrized version of QD

$$\mathcal{D}(A : B) = \min_{\{\hat{\Pi}_A^j \otimes \hat{\Pi}_B^k\}} [I(\hat{\rho}_{AB}) - I(\Phi_{AB}(\hat{\rho}_{AB}))], \quad (4)$$

where the measurement operator Φ_{AB} is given by $\Phi_{AB}(\hat{\rho}_{AB}) = \sum_{j,k} (\hat{\Pi}_A^j \otimes \hat{\Pi}_B^k) \hat{\rho}_{AB} (\hat{\Pi}_A^j \otimes \hat{\Pi}_B^k)$. Eq. (4) provides the symmetric QD considered in Ref. [21] and experimentally witnessed in Refs. [22, 23]. The vanishing of $\mathcal{D}(A : B)$ occurs if and only if the state is classical. In particular, the absence of $\mathcal{D}(A : B)$ is the key ingredient for local broadcasting, namely, local sharing of preestablished correlations [24].

Generalizations of quantum discord to multipartite states have been considered in different scenarios [18, 25–27], which intend to account for quantum correlations that may exist beyond pairwise subsystems in a composite system. In this direction, one possible approach

to account multipartite quantum correlations is to start from the symmetrized QD and then to systematically extend it to the multipartite scenario. This originates GQD as a measure of global quantum discord, as proposed in Ref. [18]. The GQD $\mathcal{D}(A_1 : \dots : A_N)$ for an arbitrary multipartite state $\hat{\rho}_{A_1 \dots A_N}$ under a set of local measurements $\{\hat{\Pi}_{A_1}^{j_1} \otimes \dots \otimes \hat{\Pi}_{A_N}^{j_N}\}$ can suitably be expressed as [18, 28, 29]

$$\mathcal{D}(A_1 : \dots : A_N) = \min_{\Phi} [I(\hat{\rho}_{A_1 \dots A_N}) - I(\Phi(\hat{\rho}_{A_1 \dots A_N}))], \quad (5)$$

where

$$\Phi(\hat{\rho}_{A_1 \dots A_N}) = \sum_k \hat{\Pi}_k \hat{\rho}_{A_1 \dots A_N} \hat{\Pi}_k, \quad (6)$$

with $\hat{\Pi}_k = \hat{\Pi}_{A_1}^{j_1} \otimes \dots \otimes \hat{\Pi}_{A_N}^{j_N}$ and k denoting the index string $(j_1 \dots j_N)$. In Eq. (5), the mutual information $I(\hat{\rho}_{A_1 \dots A_N})$ is defined following the natural generalization proposed in Ref. [20], which implies that $I(\hat{\rho}_{A_1 \dots A_N})$ and $I(\Phi(\hat{\rho}_{A_1 \dots A_N}))$ are given by

$$\begin{aligned} I(\hat{\rho}_{A_1 \dots A_N}) &= \sum_{k=1}^N S(\hat{\rho}_{A_k}) - S(\hat{\rho}_{A_1 \dots A_N}), \\ I(\Phi(\hat{\rho}_{A_1 \dots A_N})) &= \sum_{k=1}^N S(\Phi(\hat{\rho}_{A_k})) - S(\Phi(\hat{\rho}_{A_1 \dots A_N})), \end{aligned} \quad (7)$$

where

$$\Phi(\hat{\rho}_{A_k}) = \sum_{k'} \hat{\Pi}_{A_k}^{k'} \hat{\rho}_{A_k} \hat{\Pi}_{A_k}^{k'}. \quad (8)$$

GQD is symmetric with respect to subsystem exchange and shown to be non-negative for arbitrary states [18]. Moreover, it can be detected through a convenient (with no extremization procedure) witness operator [30]. Concerning its applications, GQD has been shown to be useful in the characterization of quantum phase transitions [18, 31]. In terms of operational interpretation, GQD may play a role in quantum communication, in the sense that its absence means that the quantum state simply describes a classical probability multidistribution $\sum_{i_1, \dots, i_N} p_{i_1 \dots i_N} |i_1\rangle\langle i_1| \otimes \dots \otimes |i_N\rangle\langle i_N|$ (with $p_{i_1 \dots i_N} \geq 0$, $\sum p_{i_1 \dots i_N} = 1$) and, therefore, allows for local broadcasting [24].

Monogamy of quantum correlations and global quantum discord. — Let us begin by defining the loss of correlation $\mathcal{D}_{\Phi}(A_1 : \dots : A_N)$ in a quantum state $\hat{\rho}_{A_1 \dots A_N}$ generated by a measurement $\Phi(\hat{\rho}_{A_1 \dots A_N})$, which is given by

$$\mathcal{D}_{\Phi}(A_1 : \dots : A_N) = I(\hat{\rho}_{A_1 \dots A_N}) - I(\Phi(\hat{\rho}_{A_1 \dots A_N})). \quad (9)$$

We can then show that $\mathcal{D}_{\Phi}(A_1 : \dots : A_N)$ can be decomposed in terms of loss of correlation for suitable bipartitions, as provided by the following theorem.

Theorem 1 *Given a non-selective measurement $\Phi(\hat{\rho}_{A_1 \dots A_N})$, the loss of correlation $\mathcal{D}_\Phi(A_1 : \dots : A_N)$ can be decomposed as*

$$\mathcal{D}_\Phi(A_1 : \dots : A_N) = \sum_{k=1}^{N-1} \mathcal{D}_\Phi(A_1 \dots A_k : A_{k+1}). \quad (10)$$

Proof. We rewrite $\mathcal{D}_\Phi(A_1 : \dots : A_N)$ by using Eqs. (7) and (9), which yields

$$\begin{aligned} \mathcal{D}_\Phi(A_1 : \dots : A_N) &= \sum_{k=1}^N [S(\hat{\rho}_{A_k}) - S(\Phi_k(\hat{\rho}_{A_k}))] \\ &\quad - S(\hat{\rho}_{A_1 \dots A_N}) + S(\Phi(\hat{\rho}_{A_1 \dots A_N})), \end{aligned} \quad (11)$$

We then add and subtract $S(\hat{\rho}_{A_1 \dots A_{N-1}})$ and $S(\Phi(\hat{\rho}_{A_1 \dots A_{N-1}}))$ in Eq. (11). After rearrangement of the terms, we obtain

$$\mathcal{D}_\Phi(A_1 : \dots : A_N) = \mathcal{D}_\Phi(A_1 : \dots : A_{N-1}) + \mathcal{D}_\Phi(A_1 \dots A_{N-1} : A_N). \quad (12)$$

By recursively applying Eq. (12), we can reduce the term $\mathcal{D}_\Phi(A_1 : \dots : A_{N-1})$ to a sum of bipartite contributions $\sum_{k=1}^{N-2} \mathcal{D}_\Phi(A_1 \dots A_k : A_{k+1})$ which, when added to $\mathcal{D}_\Phi(A_1 \dots A_{N-1} : A_N)$ in Eq. (12), leads to Eq. (10). ■

By taking a tripartite system ABC , Theorem 1 implies that

$$\mathcal{D}_\Phi(A : B : C) = \mathcal{D}_\Phi(A : B) + \mathcal{D}_\Phi(AB : C). \quad (13)$$

Note that Eq. (13) provides a decomposition of $\mathcal{D}_\Phi(A : B : C)$ in terms of bipartite contributions. However, as a further step to achieve a monogamy inequality, we shall relate $\mathcal{D}_\Phi(AB : C)$ with the pairwise term $\mathcal{D}_\Phi(A : C)$. This is provided by the theorem below.

Theorem 2 *The bipartite loss of correlation $\mathcal{D}_\Phi(AB : C)$ under a measurement $\Phi(\hat{\rho}_{ABC})$ cannot increase by discarding a subsystem, i.e.*

$$\mathcal{D}_\Phi(AB : C) \geq \mathcal{D}_\Phi(A : C), \quad (14)$$

provided that the conditional mutual information

$$I_\rho(AB|C) \equiv S(\rho_{AB}) + S(\rho_{AC}) - S(\rho_{ABC}) - S(\rho_A) \quad (15)$$

does not increase after $\Phi(\hat{\rho}_{ABC})$, i.e.,

$$I_\rho(AB|C) \geq I_{\Phi(\rho)}(AB|C). \quad (16)$$

Proof. By using Eq. (9) into inequality (14), we obtain

$$\begin{aligned} &S(\hat{\rho}_{AB}) - S(\hat{\rho}_{ABC}) - S(\Phi(\hat{\rho}_{AB})) + S(\Phi(\hat{\rho}_{ABC})) \\ &\geq S(\hat{\rho}_A) - S(\hat{\rho}_{AC}) - S(\Phi(\hat{\rho}_A)) + S(\Phi(\hat{\rho}_{AC})) \end{aligned} \quad (17)$$

Then, by arranging terms in inequality (17), we obtain its equivalence with the constraint (16). ■

Theorems 1 and 2 allow for the comparison between multipartite and pairwise correlations. In this direction, GQD can be derived by minimizing $\mathcal{D}_\Phi(A_1 : \dots : A_N)$ over all measurements $\Phi(\hat{\rho}_{A_1 \dots A_N})$, i.e.,

$$\mathcal{D}(A_1 : \dots : A_N) = \min_{\Phi} [\mathcal{D}_\Phi(A_1 : \dots : A_N)]. \quad (18)$$

Then, a monogamy bound for quantum correlations can be obtained, which is proved below.

Theorem 3 *For an arbitrary density operator $\hat{\rho}_{A_1 \dots A_N}$ with an amount of GQD given by $\mathcal{D}(A_1 : \dots : A_N)$, pairwise QD obeys the monogamy bound*

$$\mathcal{D}(A_1 : \dots : A_N) \geq \sum_{k=1}^{N-1} \mathcal{D}(A_1 : A_{k+1}), \quad (19)$$

provided that $I_\rho(A_1 B_k | A_{k+1}) \geq I_{\Phi(\rho)}(A_1 B_k | A_{k+1})$, for $2 \leq k \leq N-1$, with $B_k = A_2 \dots A_k$ and $\Phi(\rho)$ denoting the minimizing measurement basis for $\mathcal{D}(A_1 : \dots : A_N)$.

Proof. By requiring the conditions $I_\rho(A_1 B_k | A_{k+1}) \geq I_{\Phi(\rho)}(A_1 B_k | A_{k+1})$, for $2 \leq k \leq N-1$, we can apply Theorems 1 and 2 to obtain

$$\mathcal{D}_\Phi(A_1 : \dots : A_N) \geq \sum_{k=1}^{N-1} \mathcal{D}_\Phi(A_1 : A_{k+1}). \quad (20)$$

To obtain (20), we have identified $A \equiv A_1$, $B \equiv A_2 \dots A_k$, and $C \equiv A_{k+1}$ in Eq. (16) and then used Eq. (14). In order to obtain GQD, we minimize both sides with respect to $\Phi(\hat{\rho}_{A_1 \dots A_N})$, yielding

$$\min_{\Phi} \mathcal{D}_\Phi(A_1 : \dots : A_N) \geq \min_{\Phi} \sum_{k=1}^{N-1} \mathcal{D}_\Phi(A_1 : A_{k+1}). \quad (21)$$

However, we have that

$$\min_{\Phi} \sum_{k=1}^{N-1} \mathcal{D}_\Phi(A_1 : A_{k+1}) \geq \sum_{k=1}^{N-1} \min_{\Phi} \mathcal{D}_\Phi(A_1 : A_{k+1}). \quad (22)$$

Hence, by inserting Eq. (22) into Eq. (21), we obtain the monogamy bound stated by inequality (19). ■

In particular, for tripartite states, the monogamy inequality (19) reduces to inequality (2), with the *only* requirement that $I_\rho(AB|C) \geq I_{\Phi(\rho)}(AB|C)$, where $\Phi(\hat{\rho}_{ABC})$ denotes the minimizing basis for $\mathcal{D}(A : B : C)$.

Pure states with Schmidt decomposition. — Let us illustrate the monogamy bound (19) in the case of multipartite pure states $|\psi\rangle$ that admit Schmidt decomposition, whose explicit conditions of existence are discussed in Ref. [32]. We will assume that the system is composed by a set of qubits. In such a case, we can write $|\psi\rangle = \sum_{i=1}^2 \sqrt{p_i} |i_{A_1}\rangle \otimes \dots \otimes |i_{A_N}\rangle$, where $\{|i_{A_k}\rangle\}$ are orthonormal bases, $p_i \geq 0$, and $\sum_i p_i = 1$. For the density operator $\hat{\rho}_{A_1 \dots A_N} = |\psi\rangle\langle\psi|$, we obtain

$$\hat{\rho}_{A_1 \dots A_N} = \sum_{i,j=1}^2 \sqrt{p_i p_j} |i_{A_1} \dots i_{A_N}\rangle \langle j_{A_1} \dots j_{A_N}|. \quad (23)$$

Since Schmidt decomposition implies equal spectrum for all single-qubit reduced density operators $\hat{\rho}_{A_k}$, we obtain that $S(\hat{\rho}_{A_k}) = -\sum_{i=1}^2 p_k \log_2 p_k \equiv S$, for any individual subsystem A_k . Therefore, the mutual information is $I(\hat{\rho}_{A_1 \dots A_N}) = N S$. In order to consider measurements $\Phi(\hat{\rho}_{A_1 \dots A_N})$ over $\hat{\rho}_{A_1 \dots A_N}$, it can be shown that, by adopting projective (von Neumann) measurements, the minimization of the loss of correlation is obtained in Schmidt basis, namely, $\{\hat{\Pi}_{A_k}^{i_k}\} = \{|i_{A_k}\rangle\langle i_{A_k}|\}$. This is a consequence of both the group homomorphism of $U(2)$ to $SO(3)$ and the monotonicity of entropy under majorization (see discussion for the state $(|0 \dots 0\rangle + |1 \dots 1\rangle)/\sqrt{2}$ in Ref. [29]). Then, $\Phi(\hat{\rho}_{A_k}) = \hat{\rho}_{A_k}$, which implies $S(\Phi(\hat{\rho}_{A_k})) = S$. Moreover $\Phi(\hat{\rho}_{A_1 \dots A_N}) = \sum_{i=1}^2 p_i |i_{A_1} \dots i_{A_N}\rangle\langle i_{A_1} \dots i_{A_N}|$. Therefore, the mutual information after measurement is $I(\Phi(\hat{\rho}_{A_1 \dots A_N})) = (N-1)S$, which implies that $\mathcal{D}(A_1 : \dots : A_N) = S$. Therefore, GQD is upper limited by the von Neumann entropy S of an individual subsystem, which holds for quantum systems composed of an arbitrary number N of qubits. As an example, consider the GHZ state for N qubits, namely, $|GHZ_N\rangle = (|0 \dots 0\rangle - |1 \dots 1\rangle)/\sqrt{2}$. For this state, we obtain $\mathcal{D}(A_1 : \dots : A_N) = 1$ (independently of N) and vanishing pairwise correlations $\mathcal{D}(A_1 : A_k)$, which is in agreement with Theorem 3.

Tripartite W-GHZ mixed state. — Let us now consider a tripartite system ABC described by the W-GHZ state

$$\hat{\rho} = \lambda |W\rangle\langle W| + (1-\lambda) |GHZ\rangle\langle GHZ|, \quad (24)$$

where $|W\rangle = (|001\rangle + |010\rangle + |100\rangle)/\sqrt{3}$ and $|GHZ\rangle = (|000\rangle - |111\rangle)/\sqrt{2}$, with $0 \leq \lambda \leq 1$. Note that, for $\lambda = 0$ and $\lambda = 1$, we have pure states, given by $|GHZ\rangle$ and $|W\rangle$ states, respectively. Therefore, by adopting projective measurements, we will have that $\mathcal{D}(A : B : C) = 1$ for $\lambda = 0$. As λ increases, we numerically find out a monotonic increase of GQD until $\mathcal{D}(A : B : C) = \log_2 3$ for $\lambda = 1$. This can be seen as a consequence of the absence of Schmidt decomposition for the W state, which leaves GQD unconstrained by the entropy of an individual subsystem. For the complete range of λ , we plot GQD in Fig. 2 as well as the pairwise sum $\mathcal{D}(A : B) + \mathcal{D}(A : C)$. We observe that monogamy as given by inequality (2) is obeyed for any λ , with distinct minimizing bases $\{\Pi_{A_k}\}$ for GQD and pairwise QD. Note also from the plot that the bound is considerably tight in the sense that, for any λ , the value of GQD is sufficiently limited to ensure that the subsystems are *not* freely correlated.

Residual multipartite correlations. — The monogamy bound (2) allows for the definition of a measure \mathcal{D}_R for *residual* multipartite quantum correlations, namely, contributions to quantum correlations beyond pairwise QD. This is similar to the definition of *tangle* as a measure of residual multipartite entanglement [10]. Indeed, let us consider the particular case of tripartite permutation invariant states, such the W-GHZ state. In such a case, we

can define the residual QD as

$$\mathcal{D}_R \equiv \mathcal{D}(A : B : C) - \mathcal{D}(A : B) - \mathcal{D}(A : C). \quad (25)$$

Note that, by considering $I_\rho(AB|C) \geq I_{\Phi(\rho)}(AB|C)$ (such as in the W-GHZ state), monogamy implies that $\mathcal{D}_R \geq 0$. On the other hand, if permutation invariance is absent, the tripartite residual correlations can be defined by $\min \mathcal{D}_R$, where minimization is taken over all subsystem permutations (see Ref. [33] for a similar definition in the case of entanglement). Applying the residual measure for the GHZ state, it follows that $\mathcal{D}_R^{GHZ} = 1$, since pairwise contributions vanish. For the W state, residual QD is lower than in the case of the GHZ state, since pairwise QD is nonvanishing. Indeed, it can be shown that $\mathcal{D}^W(A : B) = \mathcal{D}^W(A : C) = 2 \log_2(3) - \frac{2}{3} - \frac{5}{6} \log_2(5) \approx 0.568$ (with the minimizing measurement found in the σ_x eigenbasis). This implies that $\mathcal{D}_R^W = \frac{4}{3} + \frac{5}{3} \log_2(5) - 3 \log_2(3) \approx 0.448$. This behavior is exhibited in Fig. 2 and is in agreement with the hierarchy found in Ref. [34].

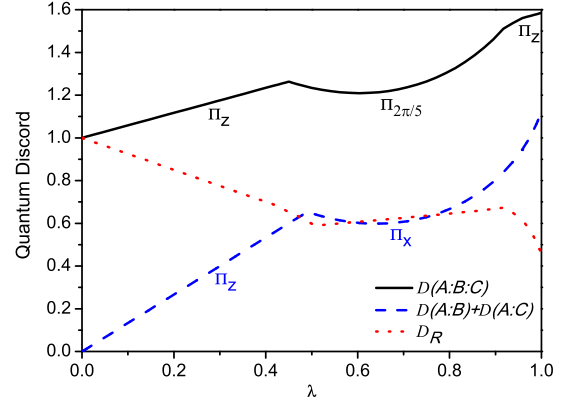


FIG. 2. (Color online) GQD, pairwise QD, and residual QD for the W-GHZ mixed state as a function of the parameter λ . For each curve, the minimizing bases $\{\Pi_{A_k}\}$ are indicated, which are equal for all qubits measured. As λ varies, the minimizing bases are denoted in terms of the Bloch sphere angles (θ, ϕ) , with Π_z , Π_x , and $\Pi_{2\pi/5}$ associated with the angles $(0, 0)$, $(\pi/2, 0)$, and $(2\pi/5, 0)$, respectively.

In conclusion, we have introduced a monogamy bound for pairwise quantum correlations based on the amount of GQD available to a multipartite system. Remarkably, this monogamy inequality holds for general states whose bipartite quantum discord is non-increasing under discard of a subsystem. In particular, oppositely to the typical monogamy bound (1), it covers both GHZ and W tripartite classes of states, providing therefore a promising setup for the investigation of measures for multipartite residual correlations.

We thank Kavan Modi and József Pitrik for helpful discussions. This work is supported by CNPq, CAPES, FAPERJ, and the Brazilian National Institute for Science and Technology of Quantum Information (INCT-IQ).

* helenacbraga@gmail.com

- [1] H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001).
- [2] V. Madhok and A. Datta, e-print arXiv:1204.6042 (2012).
- [3] M. Gu *et al.*, e-print arXiv:1203.0011 (2012).
- [4] A. Datta, A. Shaji, and C. M. Caves, Phys. Rev. Lett. **100**, 050502 (2008).
- [5] D. P. DiVincenzo *et al.*, Phys. Rev. Lett. **92**, 067902 (2004); A. Datta and S. Gharibian, Phys. Rev. A **79**, 042325 (2009); S. Boixo *et al.*, Int. J. Quantum Inf. **9**, 1643 (2011).
- [6] L. Roa, J. C. Retamal, and M. Alid-Vaccarezza, Phys. Rev. Lett. **107**, 080401 (2011).
- [7] M. S. Sarandy, Phys. Rev. A **80**, 022108 (2009); T. Werlang *et al.*, Phys. Rev. Lett. **105**, 095702 (2010).
- [8] J. Maziero *et al.*, Phys. Rev. A **80**, 044102 (2009); X. Jin-Shi *et al.*, Nat. Commun. **1**, 7 (2010).
- [9] M. Pawłowski, Phys. Rev. A **82**, 032313 (2010).
- [10] V. Coffman, J. Kundu, and W. K. Wootters, Phys. Rev. A **61**, 052306 (2000).
- [11] R. Prabhu *et al.*, Phys. Rev. A **85**, 040102(R) (2012).
- [12] G. L. Giorgi, Phys. Rev. A **84**, 054301 (2011).
- [13] X.-J. Ren and H. Fan, e-print arXiv:1111.5163 (2011).
- [14] F. F. Fanchini *et al.*, e-print arXiv:1110.1054 (2011).
- [15] Y.-K. Bai *et al.*, e-print arXiv:1206.2096 (2012).
- [16] A. Streltsov *et al.*, e-print arXiv:1112.3967 (2011).
- [17] Salini K. *et al.*, e-print arXiv:1206.4029 (2012).
- [18] C. C. Rulli and M. S. Sarandy, Phys. Rev. A **84**, 042109 (2011).
- [19] S. Luo and S. Fu, Phys. Rev. A **82**, 034302 (2010).
- [20] B. Groisman, S. Popescu, and A. Winter, Phys. Rev. A **72**, 032317 (2005).
- [21] J. Maziero, L. C. Céleri, and R. M. Serra, e-print arXiv:1004.2082 (2010).
- [22] R. Auccaise *et al.*, Phys. Rev. Lett. **107**, 070501 (2011).
- [23] G. H. Aguilar *et al.*, Phys. Rev. Lett. **108**, 063601 (2012).
- [24] M. Piani, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **100**, 090502 (2008).
- [25] K. Modi and V. Vedral, AIP Conf. Proc. **1384**, 69 (2011).
- [26] I. Chakrabarty, P. Agrawal, and A. K. Pati, Eur. Phys. J. D **65**, 605 (2011).
- [27] M. Okrasa and Z. Walczak, Europhys. Lett. **96**, 60003 (2011).
- [28] L. C. Céleri, J. Maziero, and R. M. Serra, Int. J. Quantum Inf. **9**, 1837 (2011).
- [29] J. Xu, e-print arXiv:1204.5868 (2012).
- [30] A. Saguia *et al.*, Phys. Rev. A **84**, 042123 (2011).
- [31] S. Campbell, L. Mazzola, and M. Paternostro, Int. J. Quantum Inf. **9**, 1685 (2011).
- [32] A. Peres, Phys. Lett. A **202**, 16 (1995); A. K. Pati, Phys. Lett. A **278**, 118 (2000).
- [33] G. Adesso and F. Illuminati, Phys. Rev. Lett. **99**, 150501 (2007).
- [34] G. L. Giorgi *et al.*, Phys. Rev. Lett. **107**, 190501 (2011).